

Squares and Square Roots

$$x^2 = x \cdot x$$

$$6^2 = 6 \cdot 6 = 36$$

$$15^2 = 15 \cdot 15 = 225$$

$$\sqrt{x^2} = \sqrt{x \cdot x} = x$$

$$\sqrt{49} = \sqrt{7 \cdot 7} = 7$$

$$\sqrt{400} = \sqrt{20^2} = 20$$

$$\sqrt{50} = \sqrt{2 \cdot 5 \cdot 5} = 5\sqrt{2}$$

$$\sqrt{300} = \sqrt{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}$$

$$= 2 \cdot 5 \sqrt{3}$$

$$= 10\sqrt{3}$$

Find the indicated squares.

- | | | | |
|------------|-------------|-----------------------|-----------------------|
| 1. 11^2 | 2. 8^2 | 3. 4^2 | 4. $(\frac{1}{2})^2$ |
| 5. 1.1^2 | 6. 1^2 | 7. 5^2 | 8. 6^2 |
| 9. 2.3^2 | 10. 1.2^2 | 11. $(\frac{2}{3})^2$ | 12. $(\frac{1}{4})^2$ |

Find the indicated square roots.

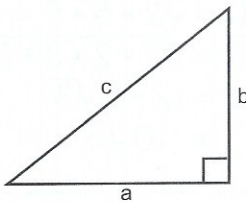
- | | | | |
|------------------|-------------------|--------------------------|---------------------------|
| 13. $\sqrt{4}$ | 14. $\sqrt{81}$ | 15. $\sqrt{100}$ | 16. $\sqrt{64}$ |
| 17. $\sqrt{121}$ | 18. $\sqrt{25}$ | 19. $\sqrt{9}$ | 20. $\sqrt{49}$ |
| 21. $\sqrt{16}$ | 22. $\sqrt{1.44}$ | 23. $\sqrt{\frac{1}{9}}$ | 24. $\sqrt{\frac{4}{25}}$ |

Simplify each of the following. Circle the answer in each row of problems that does not belong. Write the corresponding letter above the problem number below.

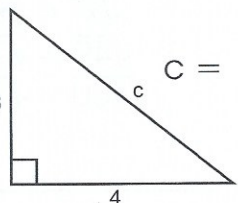
- | | | |
|---------------------|-----------------|----------------|
| 25. N $\sqrt{200}$ | L $4\sqrt{50}$ | S $2\sqrt{50}$ |
| 26. T $\sqrt{720}$ | A $12\sqrt{60}$ | Q $4\sqrt{45}$ |
| 27. R $10\sqrt{45}$ | M $3\sqrt{50}$ | P $\sqrt{450}$ |
| 28. Q $\sqrt{180}$ | A $3\sqrt{20}$ | I $2\sqrt{60}$ |
| 29. U $2\sqrt{32}$ | D $\sqrt{96}$ | W $4\sqrt{8}$ |
| 30. C $3\sqrt{18}$ | E $\sqrt{216}$ | R $2\sqrt{54}$ |

A $\sqrt{\quad}$ is truly 27 26 29 28 30 26 25 !

The Pythagorean Theorem

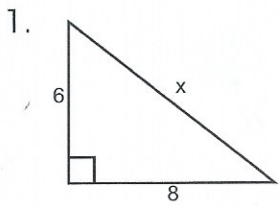


For any right Δ
 $a^2 + b^2 = c^2$
 (Pythagorean Theorem)

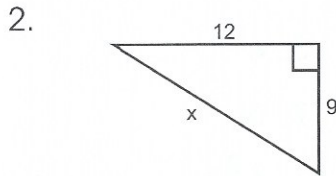


$c = \underline{\hspace{2cm}}$

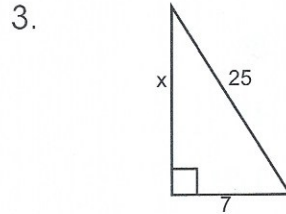
$3^2 + 4^2 = c^2$
 $9 + 16 = c^2$
 $25 = c^2$
 $5 = c$



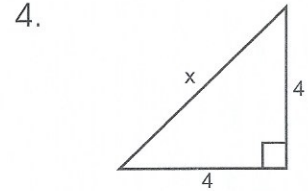
$x = \underline{\hspace{2cm}}$



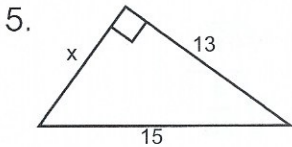
$x = \underline{\hspace{2cm}}$



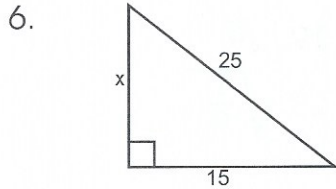
$x = \underline{\hspace{2cm}}$



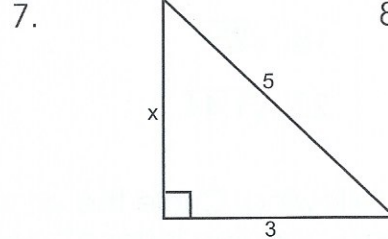
$x = \underline{\hspace{2cm}}$



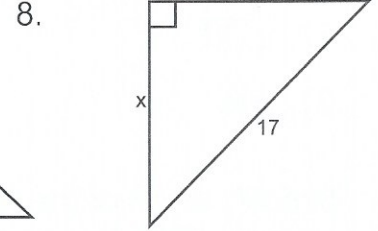
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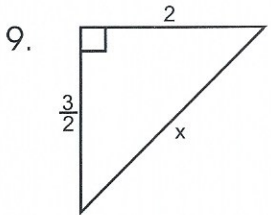
$x = \underline{\hspace{2cm}}$



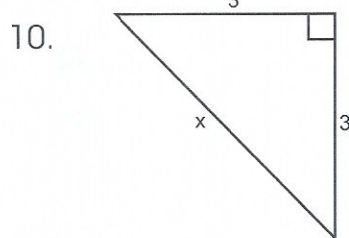
$x = \underline{\hspace{2cm}}$



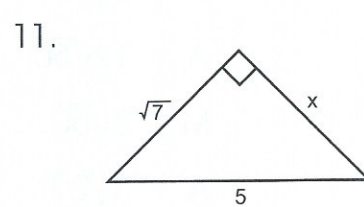
$x = \underline{\hspace{2cm}}$



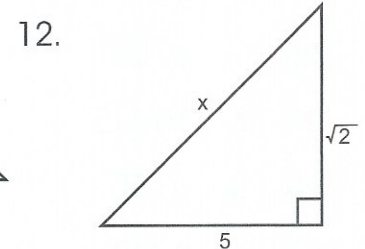
$x = \underline{\hspace{2cm}}$



$x = \underline{\hspace{2cm}}$

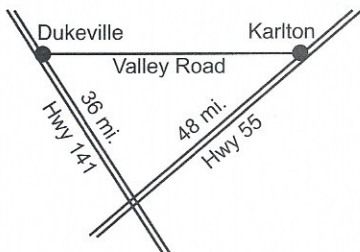


$x = \underline{\hspace{2cm}}$



$x = \underline{\hspace{2cm}}$

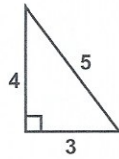
13. To go from Dukeville to Karlton, you can travel along the two main highways or the direct route along Valley Road.



- How long is the highway route?
- How long is the Valley Road route?
- How many miles do you save by taking the direct route?

Right Triangles

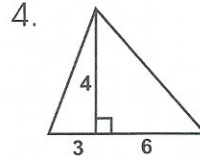
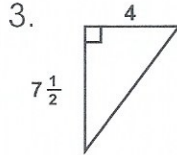
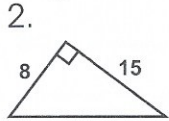
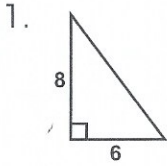
The area of a right triangle is half the product of the legs.



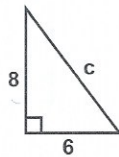
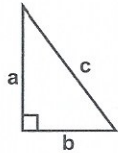
$$A = \frac{1}{2} (3) (4)$$

$$A = 6 \text{ Square units}$$

Find the areas of the right triangles below.



Pythagorean Theorem
 $a^2 + b^2 = c^2$



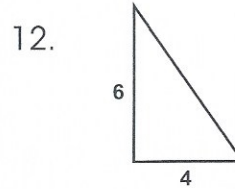
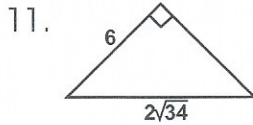
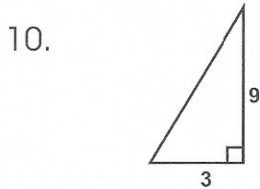
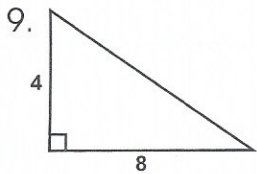
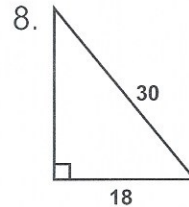
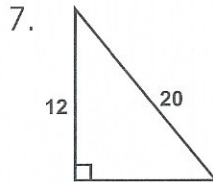
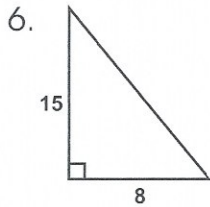
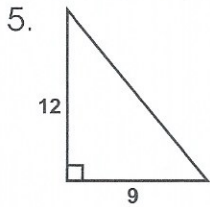
$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$10 = c$$

Find the missing side.



Cross out the correct answers each time they appear below. Use the remaining letters to complete the statement.

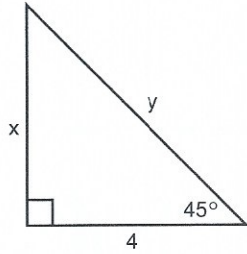
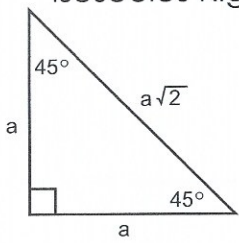
28	24	15	$10\sqrt{3}$	$3\sqrt{10}$	90	8	16	6	24
P	S	Q	Y	U	T	H	E	A	R
17	5	$4\sqrt{5}$	20	26	$2\sqrt{13}$	36	9	18	45
E	G	I	O	R	M	E	A	O	N
49	7	15	4	120	10	6	60	23	$2\sqrt{15}$
T	R	T	I	P	E	L	R	E	S

Sets of numbers like 3, 4, 5 and 5, 12, 13 are called _____.

Why is this term appropriate? _____

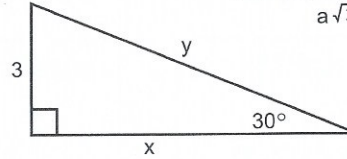
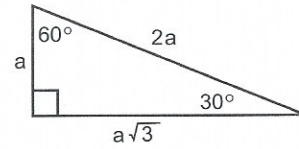
Special Right Triangles

Isosceles Right Triangle



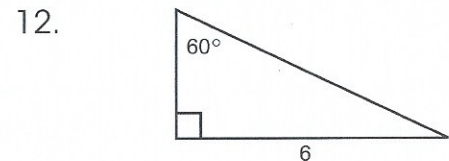
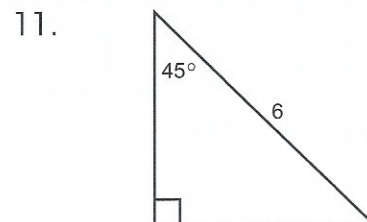
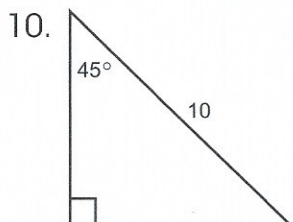
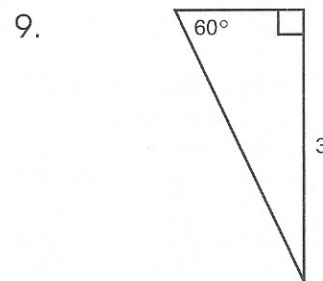
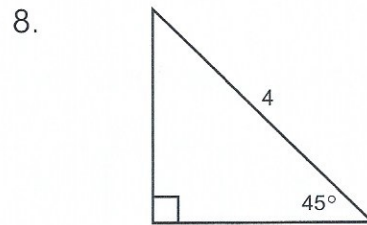
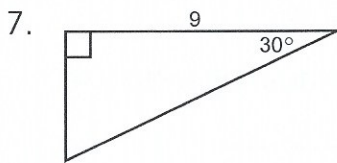
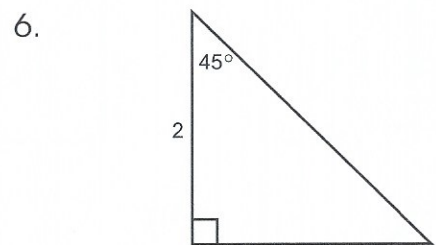
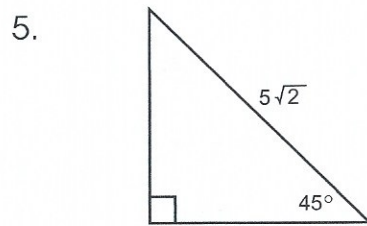
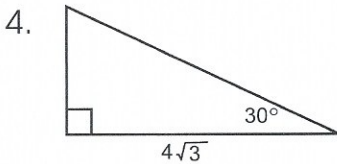
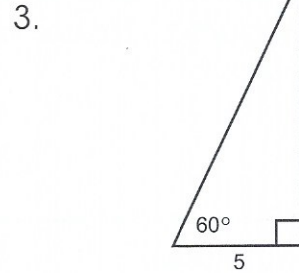
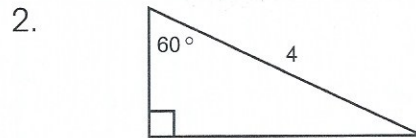
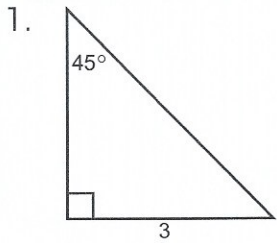
$x = 4$
 $y = 4\sqrt{2}$

30-60-90 Triangle



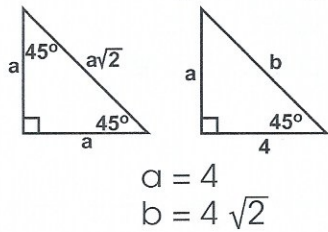
$x = 3\sqrt{3}$
 $y = 2 \cdot 3 = 6$

Find the missing sides.

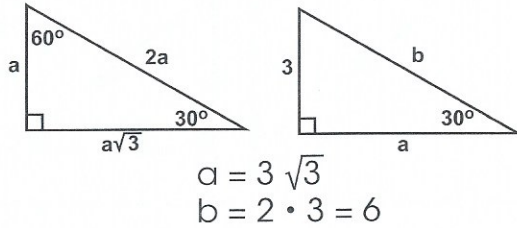


Right Triangle Challenge

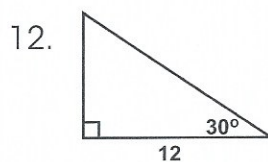
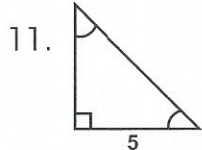
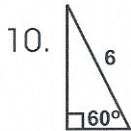
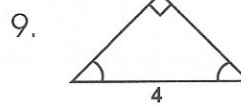
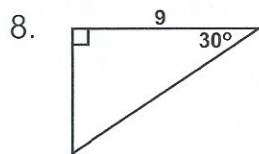
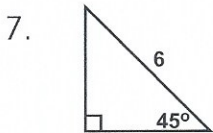
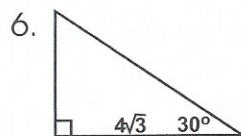
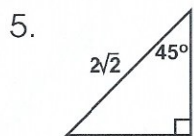
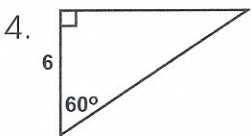
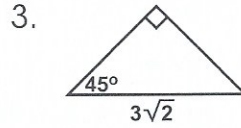
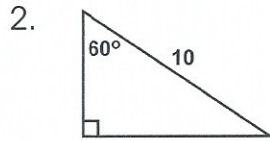
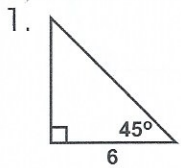
Isosceles Right Triangle



30 - 60 - 90 Triangle



Find the missing sides.

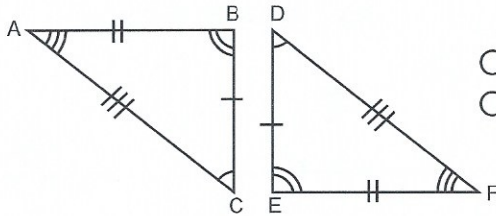


Cross out the correct answers. The remaining letters (one per space) complete the statement.

5	9	$6\sqrt{2}$	3	10	$3\sqrt{2}$	3	$4\sqrt{3}$	$3\sqrt{2}$	12	$2\sqrt{2}$
EQ	HA	UA	LT	LF	OT	HE	SQ	UA	RE	RO
$6\sqrt{3}$	$5\sqrt{3}$	25	$3\sqrt{3}$	$6\sqrt{3}$	5	20	3	$3\sqrt{3}$	36	2
OT	OF	TH	ER	AD	IU	EH	SO	FT	YP	PY
11	4	16	6	8	32	$5\sqrt{2}$	2	7	$8\sqrt{3}$	$2\sqrt{2}$
OT	TH	EN	AG	OR	US	AS	TH	E.	T.	S.

In a 30-60 degrees right triangle, the side opposite the 30-degree angle is

Parts of Congruent Triangles

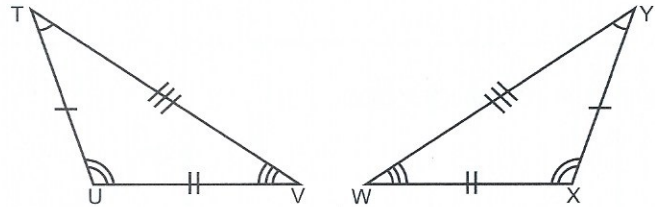


$\triangle ABC \cong \triangle FED$.
 Corresponding sides are \cong .
 Corresponding angles are \cong .

1. a. Which angle is congruent to:
 $\angle U \cong$ _____ $\angle T \cong$ _____ $\angle V \cong$ _____

b. Which side is congruent to:
 $\overline{TU} \cong$ _____ $\overline{TV} \cong$ _____ $\overline{UV} \cong$ _____

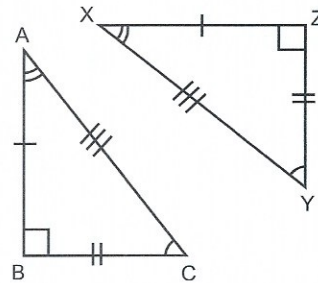
c. Write one correct congruence statement.



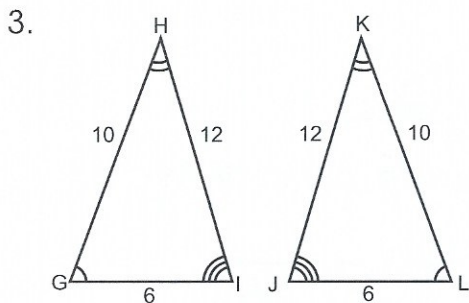
2. a. Which angle is congruent to:
 $\angle A \cong$ _____ $\angle B \cong$ _____ $\angle C \cong$ _____

b. Which side is congruent to:
 $\overline{AB} \cong$ _____ $\overline{BC} \cong$ _____ $\overline{CA} \cong$ _____

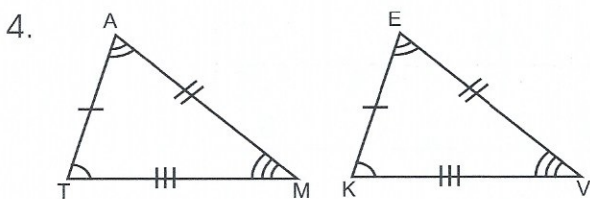
c. Write one correct congruence statement.



Complete each congruence:



- a. $\triangle HGI \cong \triangle$ _____
- b. $\triangle JKL \cong \triangle$ _____
- c. $\triangle IGH \cong \triangle$ _____
- d. $\triangle LJK \cong \triangle$ _____

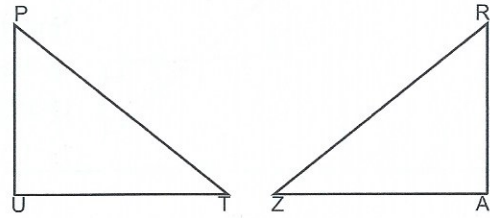


- a. $\triangle ATM \cong \triangle$ _____
- b. $\triangle TMA \cong \triangle$ _____
- c. $\triangle MAT \cong \triangle$ _____
- d. $\triangle TAM \cong \triangle$ _____

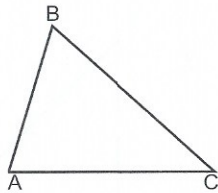
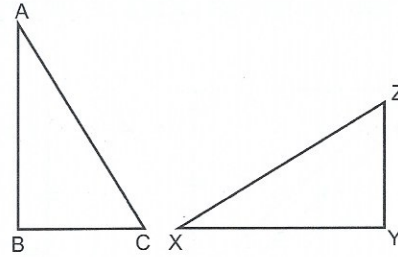
More Parts of Congruent Triangles

State whether the following appear to be true or false.

5. a. $\triangle PUT \cong \triangle RAZ$ d. $\triangle TPU \cong \triangle RZA$
 b. $\triangle TUP \cong \triangle RZA$ e. $\triangle UPT \cong \triangle ARZ$
 c. $\triangle PTU \cong \triangle ARZ$ f. $\triangle TUP \cong \triangle ZAR$



6. a. $\triangle ABC \cong \triangle XYZ$ d. $\triangle ABC \cong \triangle ZYX$
 b. $\triangle ACB \cong \triangle YXZ$ e. $\triangle BCA \cong \triangle YXZ$
 c. $\triangle CAB \cong \triangle ZXY$ f. $\triangle BAC \cong \triangle YXZ$



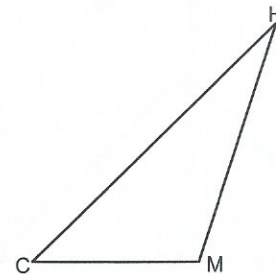
\overline{AB} is included by $\angle A$ and $\angle B$.
 $\angle C$ is included by \overline{BC} and \overline{AC} .

In $\triangle CMH$ name the angle included by each pair of sides.

7. CM and HM 8. CH and MC 9. CH and HM

In $\triangle CMH$ name the side included by each pair of angles.

10. $\angle C$ and $\angle M$ 11. $\angle H$ and $\angle C$ 12. $\angle M$ and $\angle H$

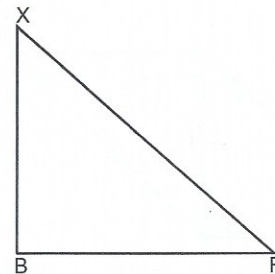


In $\triangle BFX$ name the angle included by each pair of sides.

13. BF and XF 14. BF and BX 15. FX and XB

In $\triangle BFX$ name the side included by each pair of angles.

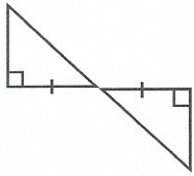
16. $\angle B$ and $\angle F$ 17. $\angle B$ and $\angle X$ 18. $\angle X$ and $\angle F$



19. If $\triangle ABC \cong \triangle DEF$, name the three pairs of corresponding sides and the three pairs of corresponding angles.

20. If $\triangle ABC \cong \triangle XYZ$, name the three pairs of corresponding sides and the three pairs of corresponding angles.

Congruent Triangles: HL

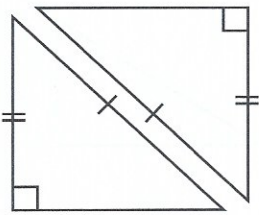


HL—Hypotenuse, Leg

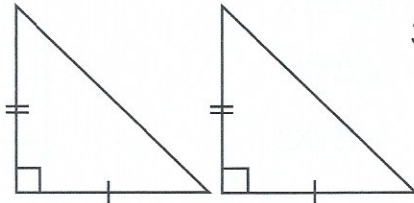
The hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another triangle $\rightarrow \cong \triangle s$.

State whether these pairs of triangles are congruent by HL. If not, write N.

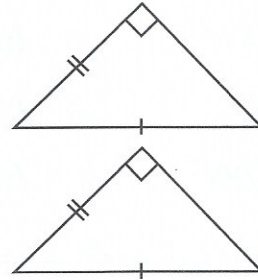
1.



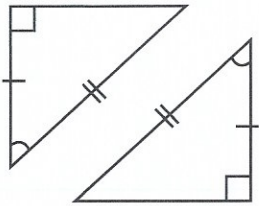
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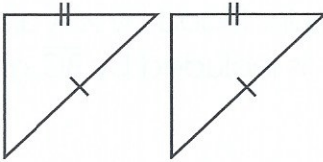
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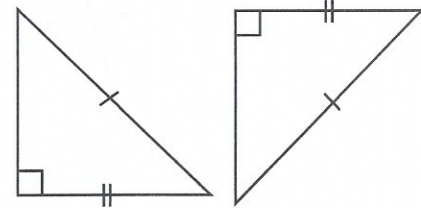
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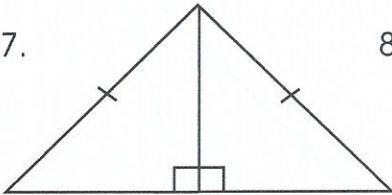
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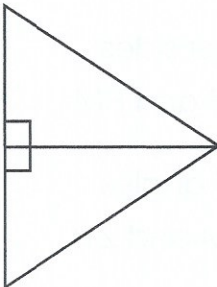
6.



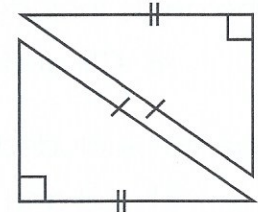
7.



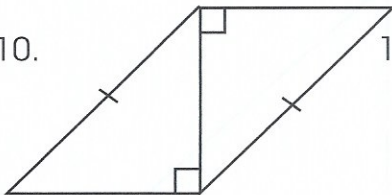
8.



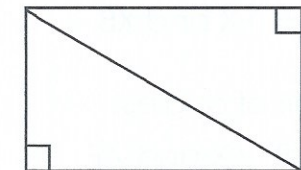
9.



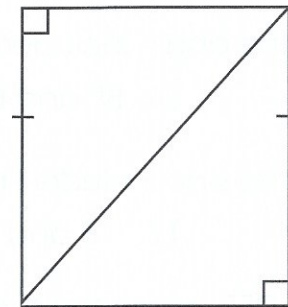
10.



11.



12.

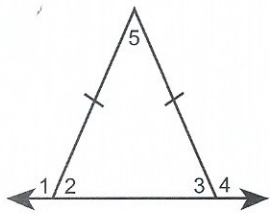
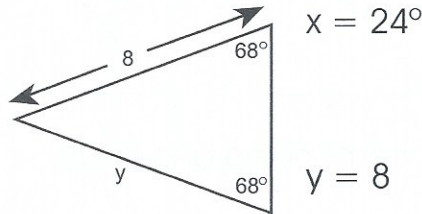


Isosceles Triangle Properties

If 2 sides of a triangle are \cong ,
the angles opposite them are \cong .

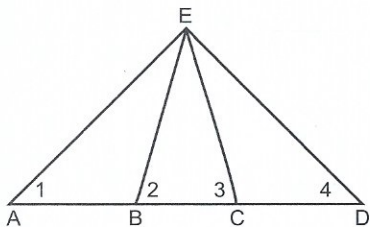


If 2 angles of a triangle are \cong ,
the sides opposite them are \cong .



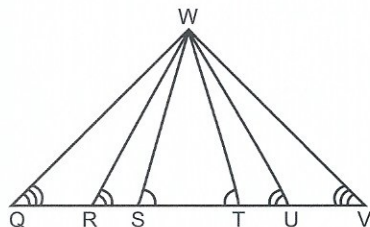
- If the $m \angle 2 = 65^\circ$, $m \angle 3 =$ _____.
- If the $m \angle 1 = 106^\circ$, $m \angle 3 =$ _____.
- If the $m \angle 1 = 109^\circ$, $m \angle 5 =$ _____.

- If the $m \angle 5 = 42^\circ$, $m \angle 2 =$ _____; $m \angle 4 =$ _____.
- If the $m \angle 4 = 121.5^\circ$, $m \angle 3 =$ _____; $m \angle 5 =$ _____.
- If the $m \angle 1 = 120^\circ$, $m \angle 3 =$ _____; $m \angle 5 =$ _____.
- If the $m \angle 3 = 55.5^\circ$, $m \angle 1 =$ _____; $m \angle 5 =$ _____.
- If the $m \angle 2 = x^\circ$, $m \angle 4 =$ _____; $m \angle 5 =$ _____.
- If the $m \angle 4 = y^\circ$, $m \angle 2 =$ _____; $m \angle 5 =$ _____.
- If the $m \angle 5 = z^\circ$, $m \angle 2 =$ _____; $m \angle 4 =$ _____.



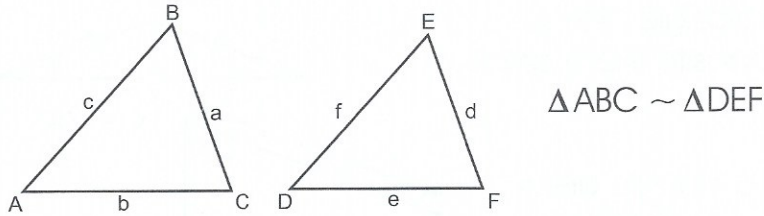
Name the correct sides to make the statements.

- If $\angle 1 \cong \angle 4$, _____ \cong _____.
- If $\angle 2 \cong \angle 3$, _____ \cong _____.
- If $m \angle 1 = 27^\circ$ and $m \angle 4 = 27^\circ$, _____ \cong _____.



- Name 3 pairs of congruent segments.
- If $m \angle QWV = 75^\circ$, $m \angle Q =$ _____.

Similar Triangles



Corresponding angles are \cong , $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$.

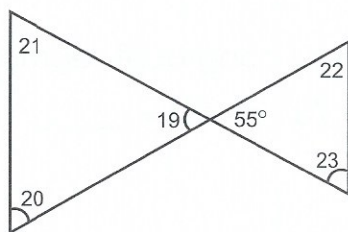
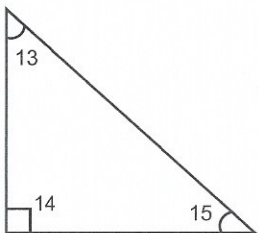
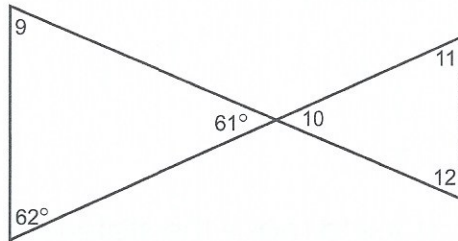
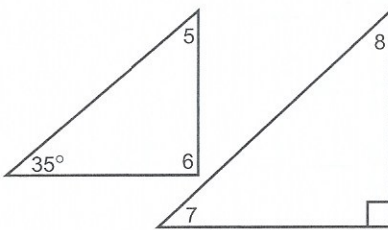
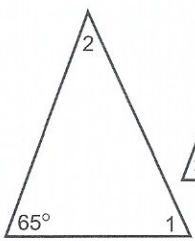
If $m\angle A = 35^\circ$ and $m\angle F = 70^\circ$, find the remaining angles.
 $m\angle D = 35^\circ$, $m\angle C = 70^\circ$, $m\angle B = 75^\circ$, $m\angle E = 75^\circ$.

Corresponding sides are in proportion. $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

If $a = 5$, $c = 9$, $f = 18$, find d . $\frac{5}{d} = \frac{9}{18}$ $9d = 90$ $d = 10$

If $e = 24$, find b . $\frac{b}{24} = \frac{9}{18}$ $18b = 216$ $b = 12$

The triangles are similar as they appear. Find the measure of angles 1-23.



1 = _____ 12 = _____

2 = _____ 13 = _____

3 = _____ 14 = _____

4 = _____ 15 = _____

5 = _____ 16 = _____

6 = _____ 17 = _____

7 = _____ 18 = _____

8 = _____ 19 = _____

9 = _____ 20 = _____

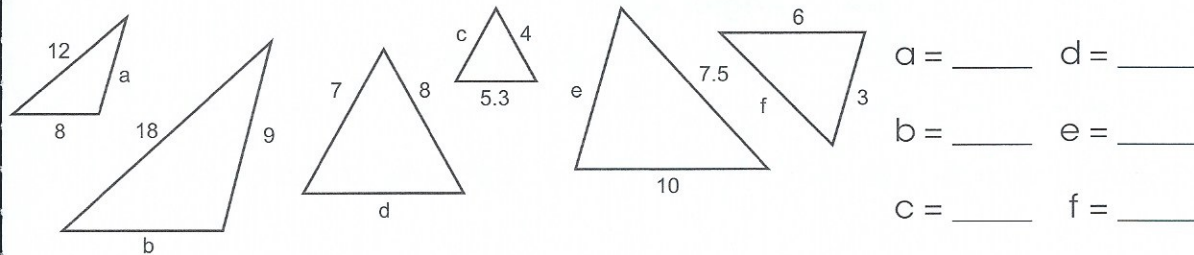
10 = _____ 21 = _____

11 = _____ 22 = _____

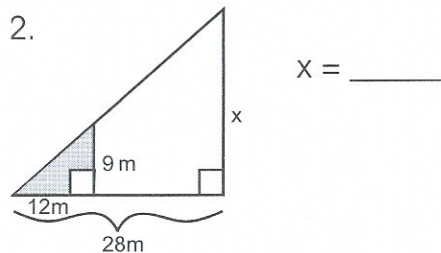
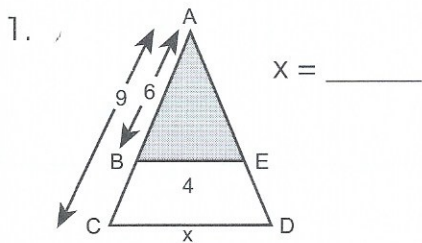
23 = _____

More Similar Triangles

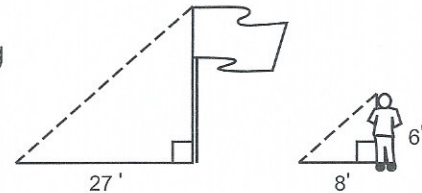
Find the lengths of sides a-f.



Find each missing measure. The triangles are similar as they appear.



3. A flagpole casts a shadow 27 feet long. A person standing nearby casts a shadow 8 feet long. If the person is 6 feet tall, how tall is the flagpole?



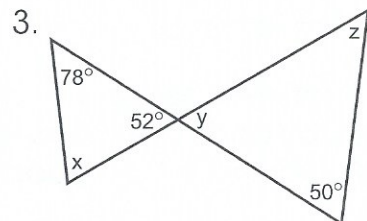
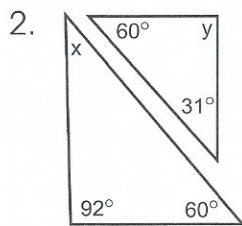
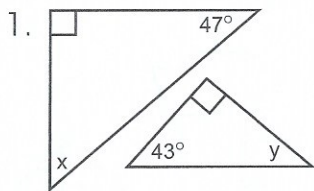
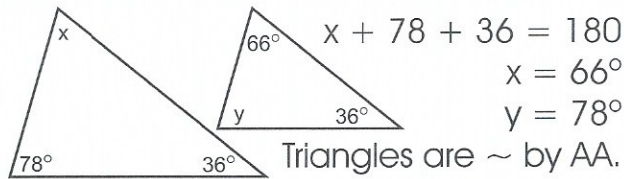
4. Christopher wants to reduce a triangular pattern with sides 16, 16, and 20 centimeters. If the longest side of the new pattern is to be 15 centimeters, how long should the other 2 sides be?
5. A 9 foot ladder leans against a building 7 feet above the ground. At what height would a 15 foot ladder touch the building if both ladders form the same angle with the ground?
6. A flagpole casts a shadow 24 feet long. A flower standing nearby casts a shadow 3 feet long. If the flagpole is 12 feet tall, how tall is the flower?
7. Sam wants to enlarge a triangle with sides 3, 6, and 6 inches. If the shortest side of the new triangle is 13 inches, how long will the other two sides be?
8. A 6 foot ladder leans against a building 4 feet above the ground. At what height would a 15 foot ladder touch the building if both ladders form the same angle with the ground?

Identifying Similar Triangles

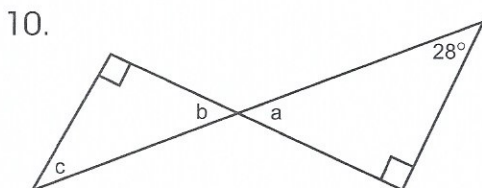
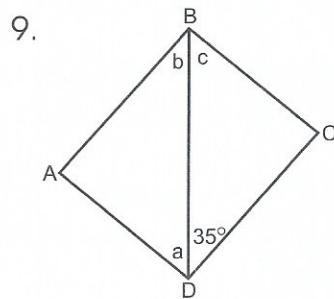
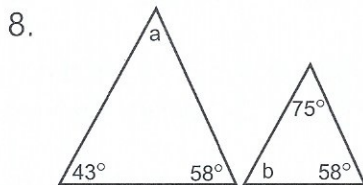
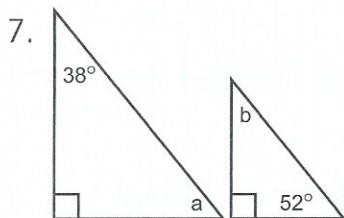
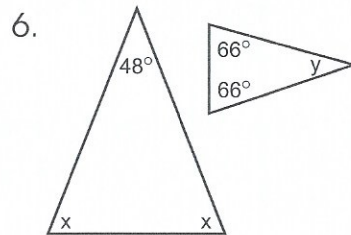
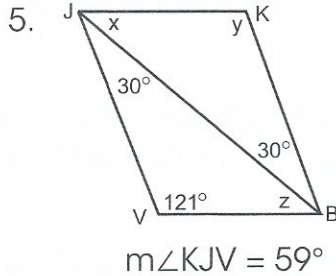
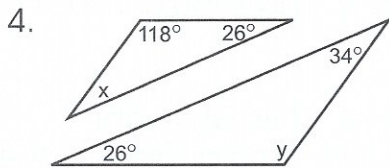
AA -- Angle, Angle

2 angles of one triangle \cong to 2 angles of another.

Find the missing angle measures. Then, tell whether the triangles are similar by the AA Property.

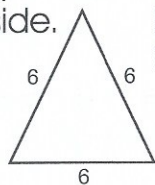


Note: Vertical angles are \cong



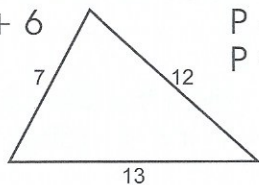
Perimeter and Similar Triangles

The perimeter of a triangle is the distance around the outside.



$$P = 6 + 6 + 6$$

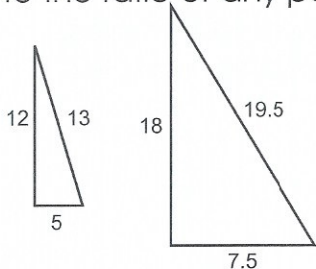
$$P = 18$$



$$P = 7 + 12 + 13$$

$$P = 32$$

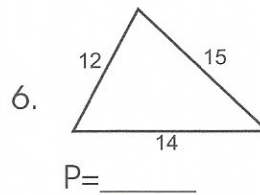
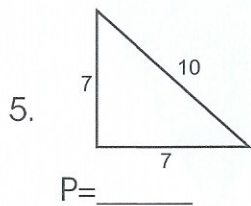
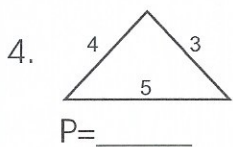
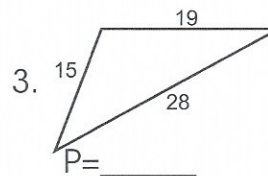
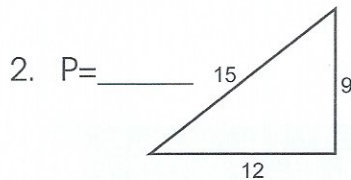
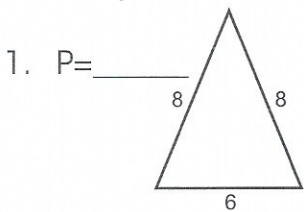
In similar triangles, the ratio of the perimeter is equal to the ratio of any pair of sides.



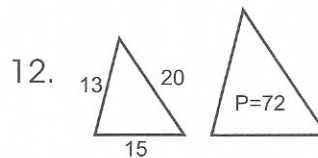
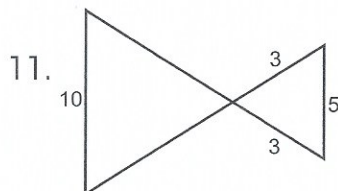
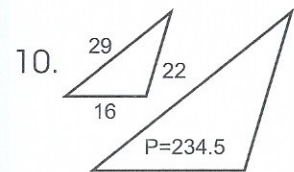
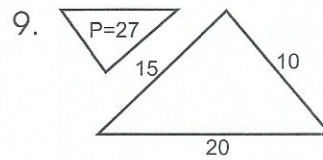
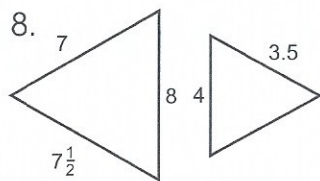
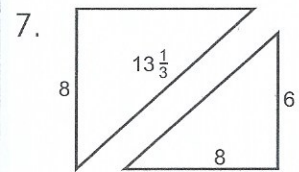
$$\frac{5}{7.5} = \frac{12}{18} = \frac{13}{19.5} = \frac{2}{3} \quad \text{Sides}$$

$$\frac{5 + 12 + 13}{7.5 + 18 + 19.5} = \frac{30}{45} = \frac{2}{3} \quad \text{Perimeter}$$

Find the perimeter.



For each pair of similar triangles, find the ratio of the perimeter of the large figure to the perimeter of the smaller.



Sierpinski Triangle

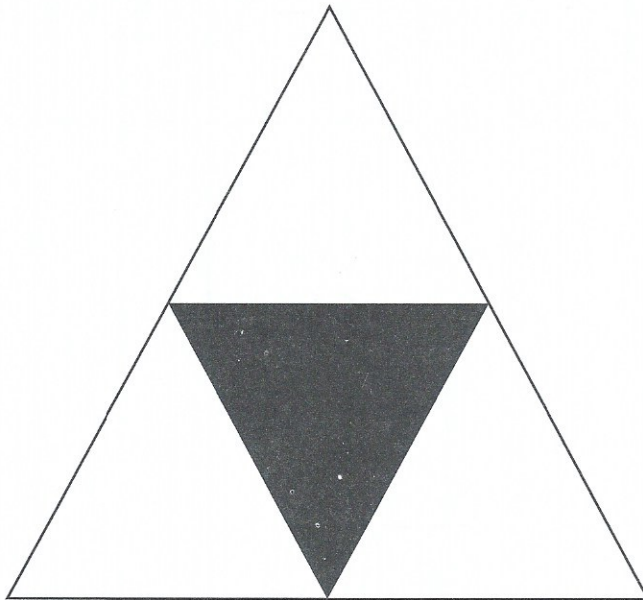
A **fractal** is a self-similar object; that is, part of the object looks like the whole object.

Below is the first stage of a fractal called the **Sierpinski Triangle**. It is created by dividing a triangle into four congruent triangles and shading the center triangle.

Stage 2: Connect the midpoints of the sides of each of the three unshaded triangles. Shade the center triangle of each section.

Stage 3: Repeat the steps of Stage 2 for each of the unshaded triangles.

Stage 4: Repeat the steps of Stage 2 for each of the unshaded triangles.

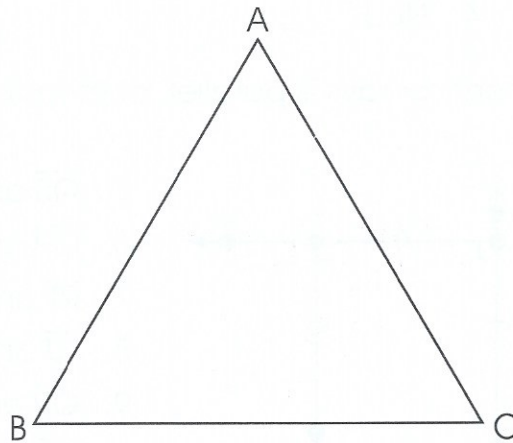


How does the number of unshaded triangles change from step to step?

A Chaotic Triangle

Work with a partner. Use a ruler and a number cube.

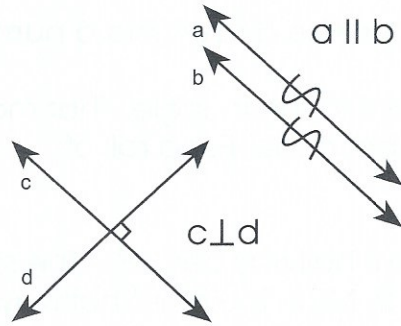
1. Mark any point in the triangle. That mark is the starting point.
2. Roll the number cube. For a roll of:
1 or 2, use vertex A
3 or 4, use vertex B
5 or 6, use vertex C
3. Mark the point halfway between the starting point and the vertex from step 2.
4. Repeat step 2. Mark the point halfway between the point marked in step 3 and the vertex determined by rolling the number cube.
5. Repeat the process approximately 100 times: roll the number cube, mark the point halfway between the last marked point and the appropriate vertex.
6. Describe the resulting pattern. _____



Parallel and Perpendicular Triangles

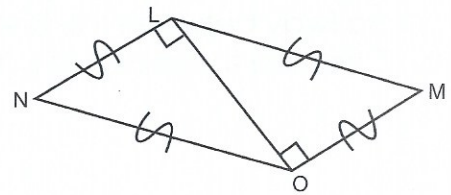
Parallel (\parallel) lines do not intersect.
(\parallel symbol denotes \parallel lines)

Perpendicular (\perp) lines intersect
to form right angles.
(\perp symbol denotes \perp lines)

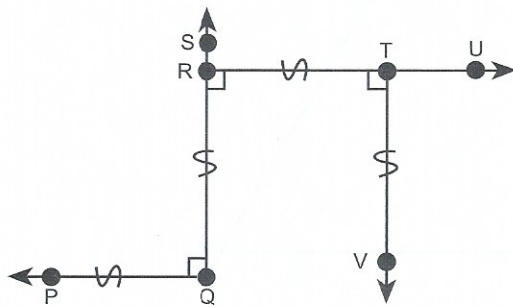


Complete.

1. $\overline{LN} \parallel$ _____
2. $\overline{ON} \parallel$ _____
3. $\overline{OM} \perp$ _____
4. $\overline{NL} \perp$ _____



Tell whether each pair of segments or rays is parallel, perpendicular, or neither.



5. \overrightarrow{QS} and \overrightarrow{TV}
6. \overrightarrow{QP} and \overrightarrow{RQ}
7. \overrightarrow{RT} and \overrightarrow{TV}
8. \overrightarrow{TU} and \overrightarrow{TV}
9. \overrightarrow{QR} and \overrightarrow{TV}
10. \overrightarrow{RU} and \overrightarrow{QP}

11. \overleftrightarrow{AC} and \overleftrightarrow{DE}
12. \overleftrightarrow{CE} and \overleftrightarrow{CF}
13. \overleftrightarrow{CF} and \overleftrightarrow{AC}
14. \overleftrightarrow{CF} and \overleftrightarrow{AD}
15. \overleftrightarrow{DF} and \overleftrightarrow{AD}
16. \overleftrightarrow{BD} and \overleftrightarrow{BC}
17. \overleftrightarrow{DF} and \overleftrightarrow{EC}
18. \overleftrightarrow{BC} and \overleftrightarrow{AD}
19. \overleftrightarrow{CE} and \overleftrightarrow{DB}
20. \overleftrightarrow{AB} and \overleftrightarrow{EF}

